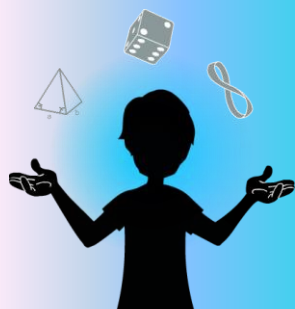# MATHEMAGIC

- DIE AND CLOCK -

AMAZINGMATHS

Materials:

- Magic trick video
- 1 die
- 1 clock

How to do the Magic Trick

Goal:

Find the value of the spectator's die.

Trick:

1. The magician stands with his back turned to the clock. He never looks at what the spectator is doing during the trick.

2. The magician asks the spectator to throw the die.

3. The magician asks the spectator to choose a number from 1 to 20.

   NOTE: The spectator does not say the number he chose to the magician.

4. The magician asks the spectator to place his finger on the clock at the value he obtained when throwing the die.

5. The magician asks the spectator to move **clockwise** on the clock a number of times equal to the number he has chosen. The spectator does not have to say where he lands on the clock. However, he must remember the result.

6. The magician asks the spectator to go back to the number of his die on the clock. He does the same thing as in step 5, but this time **counterclockwise**.

7. The magician asks the spectator to add the 2 numbers that he obtained and to tell him the result.

8. The magician announces the value of the spectator's die.

MATHEMATICAL EXPLANATION

Why This Trick Works.

To find the spectator's number, the magician uses the concept of arithmetic mean.

In fact, the spectator covered the same distance on both sides of the number to find. In the video, the spectator moves 10 positions clockwise and 10 positions counterclockwise.

Regardless of the number the spectator has chosen, he covers the same distance clockwise and counterclockwise. To better visualize this, let's use a number line.
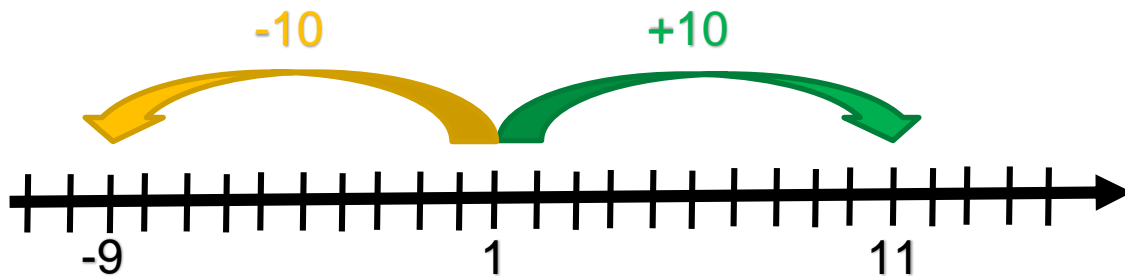
To find the point that is at the same distance from -9 and 11, we must find the **midpoint.** This point corresponds to the **arithmetic mean.**

$$\frac{-9 + 11}{2} = 1.$$

However, in this trick, we are not on a line but around a clock. The same concept applies, but with a small difficulty. On the clock, there are always 2 possible choices. For example, in the video, the 2 numbers obtained by the spectator were 3 and 11.
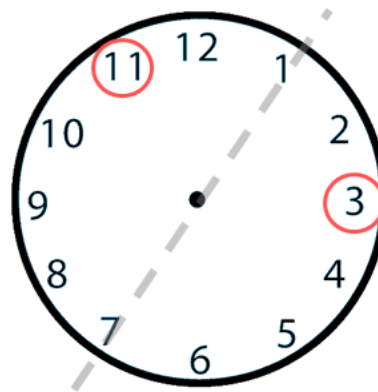
In this situation, two numbers are at the same distance from the 3 and the 11: the 1 and the 7.

In the trick, the only information given to the magician is the final sum. To average, he must simply divide the sum announced by 2. In this example, the sum if 14. The mean is then 7. However, the spectator cannot have obtained a 7 on his 6-sided die.

MATHEMATICAL EXPLANATION

As mentioned above, 7 is also a good answer, but it is not the one we are looking for. Why?

What happens when we make a movement of 12 on a clock?

A movement of 12 positions is equivalent to a movement of 0. For example, if we are on the 1 and we move 12 positions, we will be on the 1 again. So, if the sum announced by the spectator is strictly greater than 12, we must first subtract 12 and then calculate the **arithmetic mean.**

To find the value of the spectator's die, the magician simply did the following calculations:

$$\frac{14 - 12}{2} = 1$$

To go further.

The mathematical concept used above is that of "modulo".

The modulo represents the **remainder** of the whole division (also called Euclidean division). For example, the operation 16 "modulo" 3 is equal to 1. If we make the whole division of 16 by 3, we will have 16 = 3 x 5 + 1. Similarly, 7 "modulo" 2 is equal to 1.

A clock works in modulo 12. When the hour hand of a clock is on the 12, where will it be 1 hour later? One could say that it would be on the 13, since 12 + 1 = 13. Intuitively, we all know it is wrong! It is on the 1. Why? This is because 13 "modulo" 12 is equal to 1.

In the trick, what the magician has to do is take the sum of the spectator, apply the operation of "modulo" 12 and divide by 2.