MATHEMAGIC

- SQUARED -

Materials:

- Magic trick video
- 1 writing slate
- 1 pencil

How to do the Magic Trick

Goal:

Find the spectator's starting number.

Trick:

1. The magician asks the spectator to choose a number between 1 and 50 (this number will be squared so it is preferable to choose a small number if we do not want to use a calculator).
2. The magician asks the spectator to add 5 to this number and to then square this new result. The spectator's result will be number A.
3. Then, he asks to remove 5 from the starting number and to square this new result. The spectator's result will be number B.
4. The magician asks the spectator to subtract number B from number A. The spectator reveals the final result to the magician.
5. The magician is then able to say what the spectator's starting number was.

   To do so, he must simply divide the final result by 20.

MATHEMATICAL EXPLANATION

Why this trick works.

This problem can be understood algebraically. Since the starting number is unknown, let's say:

x := the starting number

After the magician has asked to add 5 to the starting number, we get the following binomial:

$x + 5.$

Calculating the square of the new number is equivalent to calculating the square of the binomial. Thus, we obtain a polynomial of degree 2, named A.

$$A = (x + 5)^2 = x^2 + 10x + 25.$$

Then, when the magician asks to remove 5 from the starting number, we get the binomial $x - 5$. Calculating the square of the new number is equivalent to calculating the square of this binomial. Thus, we obtain a polynomial of degree 2, named B.

$$B = (x - 5)^2 = x^2 - 10x + 25.$$

Next, we subtract polynomial B from polynomial A.

$$A - B$$
$$= x^2 + 10x + 25 - (x^2 - 10x + 25)$$
$$= x^2 + 10x + 25 - x^2 + 10x - 25$$
$$= 20x.$$

Remembering that x is the starting number, to find this one, the magician must simply divide the final result by 20. After all the operations are done, we obtain $20x$.

Let's take a concrete example with 24 as a starting number.

$A = (24 + 5)^2 = 24^2 + 10 \times 24 + 25.$ This is equal to $(24 + 5)^2 = 29^2 = 841.$

$B = (24 - 5)^2 = 24^2 - 10 \times 24 + 25.$ This is equal to $(24 - 5)^2 = 19^2 = 361.$

$$A - B = 24^2 + 10 \times 24 + 25 - (24^2 - 10 \times 24 + 25)$$
$$= 24^2 + 10 \times 24 + 25 - 24^2 + 10 \times 24 - 25 = 10 \times 24 + 10 \times 24 = 20 \times 24.$$

This is equal to $841 - 361 = 480 = 20 \times 24.$

So, when the magician calculates $\frac{480}{20} = \frac{20 \cdot 24}{20}$, he obtains the starting number, which is 24.