PUZZLING CARTOON

- THE MERRY-GO-ROUND -

AMAZINGMATHS

Materials:

- Video of the puzzle
- Sheets of paper
- Pencils

The puzzle

Ben works at Math-o-Fun amusement centre. Today, he is assigned to a childrens carousel. The carousel is composed of four planes that rotate at a constant speed following the trajectory of a circle. This ride lasts 5 minutes. After one ride, the centre of each airplane is shifted 75° counterclokwise compared to its position at the beginning. At 8 o'clock this morning, his boss told him: "You can take a break whenever the planes will return to their starting position."

What time will Ben take his first break at?

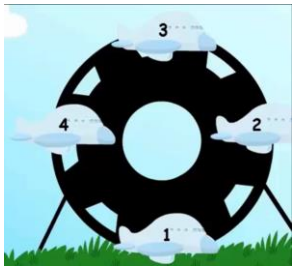
Source : *Inspired by 19e Championnat International des Jeux Mathématiques et Logiques – finale québécoise 2004-2005, Question 9, Association Québécoise des Jeux Mathématiques.*

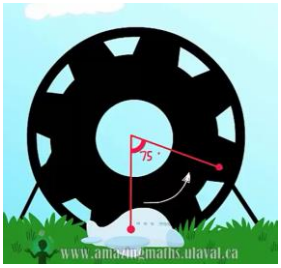
PUZZLE SOLUTION

The answer:

Ben will take his first break at 10 o'clock.

The solution:





Next time the planes will be back in this position, they will have entirely covered the circle's circumference, meaning they will have covered a total 360°. However, 360 is not a multiple of 75 since 360 ÷ 75 = 4 4/5. This information allows us to understand that, when the planes will come back in this position for the first time, the ride will be on; after one session, the centre of each plane is moved by a 75° angle compared to its position at the previous stop. A 360° movement represents 4 complete sessions and an 80% completed session (4/5). So, Ben will not be able to take his break when the planes will have entirely covered the circle's circumference for the first time.

How do we find the first moment the planes will have completed a session at the desired position?

In order to do that, we have to find the lowest common multiple (LCM) of 75 and 360. The LCM represents the smallest number of degrees needed to be covered by each plane so it completes a session at the desired position. There are many ways to find the LCM. It is possible to find it by chance, but it is also possible to find it this way:

1) Determine the prime factors of both numbers:

$$75 = 3 \times 5^2.$$

$$360 = 2^3 \times 3^2 \times 5.$$

2) Select each prime factor based on its biggest exponent in one or the other of the prime factorizations:
   - The biggest exponent for 2 is 3, so we select 2^3;
   - The biggest exponent for 3 is 2, so we select 3^2;
   - The biggest exponent for 5 is 2, so we select 5^2.
3) Determine the LCM of 75 and 360 by calculating the product of the numbers found in 2):

$$The\ LCM\ of\ 75\ and\ 360 = 2^3 \times 3^2 \times 5^2 = 1\,800.$$

We then know that the first time the planes will be back in their initial position at the end of a session, they each will have covered a 1 800° angle.

**The prime factors of a number consist in writing the said number as a product of prime numbers.*

PUZZLE SOLUTION

Since a session moves each plane by 75°, Ben will have to keep the ride on for:

$$1\,800° \div 75° = 24\ sessions.$$

A session of that ride lasts 5 minutes:

$$24\ sessions \times\ 5\ minutes = 120\ minutes.$$

$$120\ minutes\ \div\ 60\ minutes\ per\ hour = 2\ hours.$$

The planes will be back in the desired position after 2 hours. Since Ben's boss told him at 8 o'clock that he could take his break when the planes come back in this position, Ben will be able to take his break at 10 o'clock.

$$8h + 2\ hours = 10h.$$